

Supplemental Material: Temperature-dependent contact of weakly interacting single-component Fermi gases and loss rate of degenerate polar molecules

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The derivations and discussions in this supplemental material are based on the p -wave effective range theory [1, 2], which expands the effective p -wave phase shift $\delta_p(k)$ in terms of scattering volume v_p and effective range R ,

$$\frac{\tan(\delta_p(k))}{k^3} = -v_p + \frac{v_p^2}{R}k^2. \quad (\text{S1})$$

Here, k is the relative wave vector. We remind readers that we write $v_p = \Re(v_p) + i\Im(v_p)$ and similarly for R , i.e., the imaginary parts of v_p and R are $+\Im(v_p)$ and $+\Im(R)$, respectively. This convention differs some previous works [3, 4]. There are two reasons why the effective range term is explicitly accounted for in the following calculations, despite the fact that we argue in the main text that the effective range does not contribute at the temperatures where the experiments are conducted. First, it is critical for some intermediate steps. If R was dropped at the outset, some integrals would diverge. Second, Sec. II derives contributions from R and verifies our assertion presented in the main text that R leads to a T^2 term in the loss-rate coefficient. The results also allow us to connect to a conjecture made in Ref. [5].

In the main text, in order to discuss the negligible contribution of R to the thermodynamics, we assume the naturalness of v_p and R , i.e., except for the dimensionful parts, which inherit the length scale l_0 of the interaction, their dimensionless prefactor should of order 1. Because we assume that $n|v_p|$ is much smaller than 1, the characteristic length l_0 of the interaction U between particles is much smaller than the interparticle spacing $n^{1/3}$. Thus, when $n|v_p|$ goes to zero, the quantity $n^{1/3}|R|$ also goes to zero. This naturalness argument is used extensively in the discussions below.

In summary, in both the main text and this supplemental material, our calculations apply provided

$$n^{5/3} \left| \frac{v_p^2}{R} \right| \ll n|v_p| \ll 1. \quad (\text{S2})$$

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Notation List	
Symbol	Description
N	total number of particles
V	volume (homogeneous system)
n	density (homogeneous system) $n = N/V$
T_F	Fermi temperature (homogeneous system) $T_F = \frac{6^{2/3}\pi^{4/3}\hbar^2 n^{2/3}}{2mk_B}$
β	two-body loss-rate coefficient (homogeneous system) $\frac{dn}{dt} = -\beta n^2$
$n(\mathbf{r})$	local density at \mathbf{r} (harmonically trapped system)
n^{trap}	average in-situ density (harmonically trapped system) $n^{\text{trap}} = \frac{1}{N} \int d^3\mathbf{r} n(\mathbf{r})^2$
V^{trap}	in-situ volume (harmonically trapped system) $V^{\text{trap}} = N/n^{\text{trap}}$
$T_F(\mathbf{r})$	local Fermi temperature (harmonically trapped system) $T_F(\mathbf{r}) = \frac{6^{2/3}\pi^{4/3}\hbar^2 (n(\mathbf{r}))^{2/3}}{2mk_B}$
$\beta(\mathbf{r})$	local two-body loss-rate coefficient (harmonically trapped system) $\frac{dn(\mathbf{r})}{dt} = -\beta(\mathbf{r})(n(\mathbf{r}))^2$
β^{trap}	in-situ two-body loss-rate coefficient (harmonically trapped system) $\frac{dn^{\text{trap}}}{dt} = -\beta^{\text{trap}}(n^{\text{trap}})^2$
$\beta_{\text{loss}}^{\text{trap}}$	component of in-situ two-body loss-rate coefficient that is related to physical loss (harmonically trapped system) Eq. (7)
$\beta_{\text{deform}}^{\text{trap}}$	component of in-situ two-body loss-rate coefficient that is related to volume variation (harmonically trapped system) Eq. (8)
T_F^{trap}	Fermi temperature (harmonically trapped system) $T_F^{\text{trap}} = (6N)^{1/3}\hbar\omega/k_B$
T_{exp}	expansion temperature (experimental measurement) Eq. (S109)
$n_{\text{exp}}^{\text{trap}}$	in-situ average density (experimental measurement) $n_{\text{exp}}^{\text{trap}}(T_{\text{exp}}) = N\omega^3(m/\pi k_B T_{\text{exp}})^{3/2}/8$
$V_{\text{exp}}^{\text{trap}}$	in-situ volume (experimental measurement) $V_{\text{exp}}^{\text{trap}} = N/n_{\text{exp}}^{\text{trap}}$
β_{exp}	in-situ two-body loss-rate coefficient (experimental measurement) Eq. (S114)

TABLE S1. Different symbols related to densities, temperatures, and loss-rate coefficients.

The two inequalities follow from two assumptions: (1) the naturalness of v_p and R ; (2) $n|v_p| \ll 1$. For ground state $^{40}\text{K}^{87}\text{Rb}$ molecules, which are our main interest, the naturalness of v_p and (2) can be directly shown. Both the real and imaginary parts of v_p have been predicted to be about $-1.064l_0^3$ [4], where $l_0 = 118a_0$ (a_0 is the Bohr radius) is the characteristic length of the van der Waals potential. In the experiment, the typical (average in-situ) density of the system is $\sim 10^{12}\text{cm}^{-3}$; thus $n|v_p| \sim 3.7 \times 10^{-7} \ll 1$. The effective range R of $^{40}\text{K}^{87}\text{Rb}$ has, to the best of our knowledge, neither been calculated nor measured. Nevertheless, measurements of R for other atomic ultracold gases provide an indirect argument for verifying the hypothesis. For ground state ^{40}K atoms, $v_p \simeq (96.74a_0)^3$ and $R \simeq 46.22a_0$ [6, 7]. If one treats $|v_p|^{1/3}$ as the length scale of the potential, the coefficient of R is ~ 0.48 , which is of order 1.

The main text as well as Secs. IV and V of this supplemental material use a variety of temperatures, densities, and loss-rate coefficients. The notation is summarized in Table S1.

I. LINDBLAD EQUATION AND LOSS RELATION

For a Hermitian Hamiltonian \hat{H} , the density matrix in the Schrödinger picture follows the von Neumann-equation

$$i\hbar \frac{\partial \rho}{\partial t} = [\hat{H}, \rho], \quad (\text{S3})$$

where $\rho = |\psi\rangle\langle\psi|$ is the density matrix and $|\psi\rangle$ the state vector in the Schrödinger picture. This section shows that the von Neumann-equation takes on a different form for a non-Hermitian Hamiltonian \hat{H}_{eff} . Using the density matrix framework, we give a detailed derivation of Eq. (2) in the main text.

We start by separating the effective non-Hermitian Hamiltonian \hat{H}_{eff} , Eq. (1), of the homogeneous system into its real and imaginary parts,

$$\hat{H}_{\text{eff}} = \Re(\hat{H}_{\text{eff}}) + i\Im(\hat{H}_{\text{eff}}). \quad (\text{S4})$$

Using the fermionic field operators $\psi(\mathbf{r})$ and $\psi^\dagger(\mathbf{r})$ and accounting for two-body interactions and losses through the complex potential U , the real and imaginary parts read

$$\Re(\hat{H}_{\text{eff}}) = \int d^3r \Psi^\dagger(\mathbf{r}) \left(-\frac{\hbar^2 \nabla_{\mathbf{r}}^2}{2m} \right) \Psi(\mathbf{r}) + \frac{1}{2} \int d^3r d^3r' \Re(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}) \quad (\text{S5})$$

$$\Im(\hat{H}_{\text{eff}}) = \frac{1}{2} \int d^3r d^3r' \Im(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}). \quad (\text{S6})$$

The time-dependent Schrödinger equation reads

$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = (\Re(\hat{H}_{\text{eff}}) + i\Im(\hat{H}_{\text{eff}})) |\psi\rangle. \quad (\text{S7})$$

Taking the adjoint of Eq. (S7), we obtain

$$-i\hbar \frac{\partial \langle\psi|}{\partial t} = \langle\psi| (\Re(\hat{H}_{\text{eff}}) - i\Im(\hat{H}_{\text{eff}})). \quad (\text{S8})$$

Multiplying Eq. (S7) from the right with $\langle\psi|$ and Eq. (S8) from the left with $|\psi\rangle$, one obtains

$$i\hbar \frac{\partial \langle\psi|}{\partial t} \langle\psi| = (\Re(\hat{H}_{\text{eff}}) + i\Im(\hat{H}_{\text{eff}})) |\psi\rangle \langle\psi|, \quad (\text{S9})$$

$$-i\hbar |\psi\rangle \frac{\partial \langle\psi|}{\partial t} = |\psi\rangle \langle\psi| (\Re(\hat{H}_{\text{eff}}) - i\Im(\hat{H}_{\text{eff}})). \quad (\text{S10})$$

The "von Neumann-equation" for the non-Hermitian Hamiltonian \hat{H}_{eff} is obtained by subtracting Eq. (S10) from Eq. (S9):

$$i\hbar \frac{\partial \rho}{\partial t} = [\Re(\hat{H}_{\text{eff}}), \rho] + i\{\Im(\hat{H}_{\text{eff}}), \rho\}. \quad (\text{S11})$$

Equation (S11) is a "problematic master equation," because it does not conserve the trace of the density matrix. Specifically, one finds that the change of the trace of ρ with time is governed by the imaginary part of the effective Hamiltonian,

$$\frac{\partial(\text{Tr}\rho)}{\partial t} = \frac{2\text{Tr}(\Im(\hat{H}_{\text{eff}})\rho)}{\hbar}. \quad (\text{S12})$$

To fix the loss of probability, the Lindblad term $-i\sum_n L_n \rho L_n^\dagger$, where L_n is the Lindblad jump operator, can be added to Eq. (S11) to change it to a Lindblad equation:

$$i\hbar \frac{\partial \rho}{\partial t} = [\Re(\hat{H}_{\text{eff}}), \rho] + i\{\Im(\hat{H}_{\text{eff}}), \rho\} - i\sum_n L_n \rho L_n^\dagger. \quad (\text{S13})$$

The jump operators are determined by requiring that the trace of ρ does not change with time:

$$\frac{\partial(\text{Tr}\rho)}{\partial t} = \frac{2\text{Tr}(\Im(\hat{H}_{\text{eff}})\rho) - \sum_n \text{Tr}(L_n \rho L_n^\dagger)}{\hbar}. \quad (\text{S14})$$

It follows

$$\text{Tr} \left(2\Im(\hat{H}_{\text{eff}})\rho - \sum_n L_n^\dagger L_n \rho \right) = 0 \quad (\text{S15})$$

or

$$\Im(\hat{H}_{\text{eff}}) = \frac{1}{2} \sum_n L_n^\dagger L_n. \quad (\text{S16})$$

Substituting Eq. (S16) into Eq. (S13), one gets a master equation of the conventional Lindblad form,

$$i\hbar \frac{\partial \rho}{\partial t} = [\Re(\hat{H}_{\text{eff}}), \rho] - \frac{i}{2} \sum_n (L_n^\dagger L_n \rho + \rho L_n^\dagger L_n - 2L_n \rho L_n^\dagger). \quad (\text{S17})$$

An explicit expression for L_n can be read off by comparing Eq. (S16) with Eq. (S6):

$$\sum_n \rightarrow \int d^3 r d^3 r', \quad (\text{S18})$$

$$L_n \rightarrow L_{|\mathbf{r}-\mathbf{r}'|} = \sqrt{\Im(U(|\mathbf{r}-\mathbf{r}'|))} \Psi(\mathbf{r}') \Psi(\mathbf{r}). \quad (\text{S19})$$

In second quantization, the total number of particles N is given by the number operator \hat{N} ,

$$\hat{N} = \int d^3r \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}). \quad (\text{S20})$$

Its expectation value is given by $\langle \hat{N} \rangle = \text{Tr}(\rho \hat{N})$, where ρ depends on the temperature T . Multiplying Eq. (S13) by \hat{N} , subsequently taking the trace, and using that $\partial \hat{N} / \partial t$ is equal to zero in the Schrödinger picture, we obtain

$$i\hbar \frac{\partial \langle \text{Tr}(\rho \hat{N}) \rangle}{\partial t} = \text{Tr}([\Re(\hat{H}_{\text{eff}}), \rho] \hat{N}) + i\text{Tr}(\{\Im(\hat{H}_{\text{eff}}), \rho\} \hat{N}) - i\text{Tr}\left(\sum_n L_n \rho L_n^\dagger \hat{N}\right). \quad (\text{S21})$$

Note that the first term on the right hand side of Eq. (S21) is zero due to the commutativity between \hat{N} and \hat{H}_{eff} :

$$\begin{aligned} \text{Tr}([\Re(\hat{H}_{\text{eff}}), \rho] \hat{N}) &= \text{Tr}(\Re(\hat{H}_{\text{eff}}) \rho \hat{N}) - \text{Tr}(\rho \Re(\hat{H}_{\text{eff}}) \hat{N}) = \text{Tr}(\hat{N} \Re(\hat{H}_{\text{eff}}) \rho) - \text{Tr}(\Re(\hat{H}_{\text{eff}}) \hat{N} \rho) \\ &= \text{Tr}([\hat{N}, \Re(\hat{H}_{\text{eff}})] \rho) = 0. \end{aligned} \quad (\text{S22})$$

Thus, Eq. (S21) reduces to

$$\hbar \frac{\partial \langle \hat{N} \rangle}{\partial t} = \text{Tr}(\{\hat{N}, \Im(\hat{H}_{\text{eff}})\} \rho) - \text{Tr}\left(\sum_n L_n^\dagger \hat{N} L_n \rho\right). \quad (\text{S23})$$

Rearranging the field operators in the expressions that appear on the right hand side of Eq. (S23) to be normal ordered,

$$\begin{aligned} \{\hat{N}, \Im(\hat{H}_{\text{eff}})\} &= \int d^3r \int d^3r' \int d^3r'' \Im(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi^\dagger(\mathbf{r}'') \Psi(\mathbf{r}'') \Psi(\mathbf{r}') \Psi(\mathbf{r}) \\ &\quad + 2 \int d^3r \int d^3r' \Im(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}), \end{aligned} \quad (\text{S24})$$

$$\sum_n L_n^\dagger \hat{N} L_n = \int d^3r \int d^3r' \int d^3r'' \Im(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi^\dagger(\mathbf{r}'') \Psi(\mathbf{r}'') \Psi(\mathbf{r}') \Psi(\mathbf{r}), \quad (\text{S25})$$

Eq. (S23) simplifies significantly:

$$\hbar \frac{\partial \langle \hat{N} \rangle}{\partial t} = 2\text{Tr}\left(\int d^3r \int d^3r' \Im(U(|\mathbf{r} - \mathbf{r}'|)) \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}) \rho\right). \quad (\text{S26})$$

The right-hand side of Eq. (S26) can be rewritten compactly in terms of the imaginary part of the effective Hamiltonian:

$$\hbar \frac{\partial \langle \hat{N} \rangle}{\partial t} = 4\langle \Im(\hat{H}_{\text{eff}}) \rangle; \quad (\text{S27})$$

While we mostly work with the homogeneous system, we note that external potentials can be readily added to Eq. (1) because only U contributes to the imaginary part in the above equation.

Since we are interested in determining losses, our task is to derive an explicit expression for $\langle \Im(\hat{H}_{\text{eff}}) \rangle = \text{Tr}(\rho \Im(\hat{H}_{\text{eff}}))$ for a weakly-interacting p -wave gas. Within the p -wave effective range theory, U is a function of v_p and R . Correspondingly, the operator \hat{H}_{eff} is also a function of v_p and R : $\hat{H}_{\text{eff}} = \hat{H}_{\text{eff}}(v_p, R)$. When the absolute values of the dimensionless quantities $\mathcal{V} = nv_p$ and $\mathcal{R} = n^{1/3}R$ are both small, one can obtain $\Im(\hat{H}_{\text{eff}})$ by analytically continuing $\hat{H}_{\text{eff}}(\mathcal{V}, \mathcal{R})$ around $(\Re(\mathcal{V}), \Re(\mathcal{R}))$. Treating \hat{H}_{eff} as a function of the dimensionless quantities \mathcal{V} and \mathcal{R} , the analytic continuation reads

$$\Im(\hat{H}_{\text{eff}}(\mathcal{V}, \mathcal{R})) = \left. \frac{\partial \hat{H}_{\text{eff}}(\Re(\mathcal{V}), \Re(\mathcal{R}))}{\partial \mathcal{V}} \right|_{\mathcal{R}} \Im(\mathcal{V}) + \left. \frac{\partial \hat{H}_{\text{eff}}(\Re(\mathcal{V}), \Re(\mathcal{R}))}{\partial \mathcal{R}} \right|_{\mathcal{V}} \Im(\mathcal{R}) \quad (\text{S28})$$

or

$$\Im(\hat{H}_{\text{eff}}(\mathcal{V}, \mathcal{R})) = \left. \frac{\partial \hat{H}_{\text{eff}}(\Re(v_p), \Re(R))}{\partial v_p} \right|_R \Im(v_p) + \left. \frac{\partial \hat{H}_{\text{eff}}(\Re(v_p), \Re(R))}{\partial R} \right|_{v_p} \Im(R). \quad (\text{S29})$$

It is important to note that the arguments of \hat{H}_{eff} on the right hand sides of Eqs. (S28) and (S29) are the *real parts* of the dimensionless quantities \mathcal{V} and \mathcal{R} and the *real parts* of the dimensionful scattering volume v_p and effective range R , respectively.

Because we are concerned with the variation of the total number of particles at finite temperature, it is convenient to work in the grand canonical ensemble. The partition function \mathcal{Z} and thermal state density matrix ρ are given by [8, 9]

$$\mathcal{Z} = \exp\left(-\frac{\Omega}{k_B T}\right) = \text{Tr}\left(\exp\left[\frac{-(\hat{H}_{\text{eff}} - \mu\hat{N})}{k_B T}\right]\right), \quad (\text{S30})$$

$$\rho = \exp\left[\frac{\Omega - (\hat{H}_{\text{eff}} - \mu\hat{N})}{k_B T}\right], \quad (\text{S31})$$

where Ω and μ are the grand potential and chemical potential, respectively. In what follows, we will prove that

$$\langle \mathfrak{S}(\hat{H}_{\text{eff}}(v_p, R)) \rangle = \frac{\partial F(\mathfrak{R}(v_p), \mathfrak{R}(R))}{\partial v_p} \Big|_R \mathfrak{S}(v_p) + \frac{\partial F(\mathfrak{R}(v_p), \mathfrak{R}(R))}{\partial R} \Big|_{v_p} \mathfrak{S}(R) \quad (\text{S32})$$

holds in the weak interaction limit. In Eq. (S32), $F = \Omega + \mu\langle\hat{N}\rangle$ denotes the Helmholtz free energy. Equation (S32) is important since it will subsequently allow us to relate the partial derivatives of the Helmholtz free energy to p -wave contacts. Treating—consistent with our effective p -wave theory framework— Ω and μ as functions of v_p and R , we write

$$\frac{\partial F}{\partial v_p} \Big|_R = \frac{\partial(\Omega + \mu\langle\hat{N}\rangle)}{\partial v_p} \Big|_R = \frac{\partial\Omega}{\partial v_p} \Big|_R + \frac{\partial\mu}{\partial v_p} \Big|_R \langle\hat{N}\rangle, \quad (\text{S33})$$

$$\frac{\partial F}{\partial R} \Big|_{v_p} = \frac{\partial(\Omega + \mu\langle\hat{N}\rangle)}{\partial R} \Big|_{v_p} = \frac{\partial\Omega}{\partial R} \Big|_{v_p} + \frac{\partial\mu}{\partial R} \Big|_{v_p} \langle\hat{N}\rangle. \quad (\text{S34})$$

Applying the definitions from Eqs. (S30) and (S31), the partial derivative of the grand potential with respect to v_p while holding R constant can be related to that of the effective Hamiltonian and the chemical potential with respect to the same variable, taken also while holding R constant:

$$\begin{aligned} \frac{\partial\Omega}{\partial v_p} \Big|_R &= -k_B T \frac{\partial \ln \mathcal{Z}}{\partial v_p} \Big|_R \\ &= -\frac{k_B T}{\mathcal{Z}} \frac{\partial}{\partial v_p} \left[\text{Tr} \left(\exp \left(\frac{-\hat{H}_{\text{eff}} + \mu\hat{N}}{k_B T} \right) \right) \right] \Big|_R \\ &= \text{Tr} \left[\exp \left(\frac{\Omega - \hat{H}_{\text{eff}} + \mu\hat{N}}{k_B T} \right) \frac{\partial \hat{H}_{\text{eff}}}{\partial v_p} \Big|_R \right] \\ &\quad - \text{Tr} \left[\exp \left(\frac{\Omega - \hat{H}_{\text{eff}} + \mu\hat{N}}{k_B T} \right) \hat{N} \right] \frac{\partial\mu}{\partial v_p} \Big|_R \\ &= \left\langle \frac{\partial \hat{H}_{\text{eff}}}{\partial v_p} \Big|_R \right\rangle - \langle \hat{N} \rangle \frac{\partial\mu}{\partial v_p} \Big|_R. \end{aligned} \quad (\text{S35})$$

Similarly, one can find

$$\frac{\partial\Omega}{\partial R} \Big|_{v_p} = \left\langle \frac{\partial \hat{H}_{\text{eff}}}{\partial R} \Big|_{v_p} \right\rangle - \langle \hat{N} \rangle \frac{\partial\mu}{\partial R} \Big|_{v_p}. \quad (\text{S36})$$

Combining Eqs. (S33)-(S36) and (S29), we obtain Eq. (S32); this completes the proof of Eq. (S32).

As already alluded to above, the next step is to relate the partial derivatives in Eq. (S32) that involve the Helmholtz

free energy F to p -wave contacts. The p -wave contacts are defined through the following equations [7, 10, 11]:

$$\left. \frac{\partial F}{\partial v_p^{-1}} \right|_R = -\frac{\hbar^2}{2m} \sum_m C_v^{(m)}, \quad (\text{S37})$$

$$\left. \frac{\partial F}{\partial R^{-1}} \right|_{v_p} = -\frac{\hbar^2}{2m} \sum_m C_R^{(m)}, \quad (\text{S38})$$

where the superscript $m = -1, 0, 1$ represents the three p -wave scattering sub-channels. For a system with isotropic two-body interactions—as assumed throughout this work—the sub-channel contacts are equal to each other: $C_v^{(m)} = C_v$ and $C_R^{(m)} = C_R$. We thus have

$$\left. \frac{\partial F}{\partial v_p^{-1}} \right|_R = -\frac{3\hbar^2}{2m} C_v, \quad (\text{S39})$$

$$\left. \frac{\partial F}{\partial R^{-1}} \right|_{v_p} = -\frac{3\hbar^2}{2m} C_R. \quad (\text{S40})$$

Changing the variables of the derivatives from v_p^{-1} and R^{-1} to v_p and R , the partial derivatives of F in Eq. (S32) can be expressed in terms of the p -wave contacts:

$$\begin{aligned} \frac{dN}{dt} &= \frac{4}{\hbar} \left(\left. \frac{\partial F}{\partial v_p} \right|_R \mathfrak{S}(v_p) + \left. \frac{\partial F}{\partial R} \right|_{v_p} \mathfrak{S}(R) \right) \\ &= -\frac{4}{\hbar} \left(\frac{1}{(\Re(v_p))^2} \left. \frac{\partial F}{\partial v_p^{-1}} \right|_R \mathfrak{S}(v_p) + \frac{1}{(\Re(R))^2} \left. \frac{\partial F}{\partial R^{-1}} \right|_{v_p} \mathfrak{S}(R) \right) \\ &= \frac{6\hbar}{m} \left(C_v \frac{\mathfrak{S}(v_p)}{(\Re(v_p))^2} + C_R \frac{\mathfrak{S}(R)}{(\Re(R))^2} \right). \end{aligned} \quad (\text{S41})$$

II. SECOND-ORDER VIRIAL EXPANSION FOR HOMOGENEOUS AND TRAPPED SYSTEMS

This section is devoted to explaining the second-order virial expansion applied in this work. The virial expansion is applicable when the fugacity $z = \exp(\mu/k_B T)$ is small compared to 1. This condition is equivalent to demanding that $n\lambda^3$, where λ denotes the thermal wave length (see below for its definition), is small compared to 1. This condition is fulfilled at high temperatures. Taylor-expanding the grand potential Ω up to second order in z , the virial expansion for the grand potential Ω and the expression for the second virial coefficient b_2 of the interacting system read [12]

$$\Omega = -k_B T Q_1 (z + b_2 z^2), \quad (\text{S42})$$

$$b_2 = \frac{Q_2 - Q_1^2/2}{Q_1}. \quad (\text{S43})$$

Here, Q_n denotes the canonical partition function of the n -body system.

Homogeneous system.— To calculate b_2 , it is convenient to divide it into two terms, namely $b_2 = b_2^{(0)} + \Delta b_2$, where $b_2^{(0)}$ is the second virial coefficient of the non-interacting system ($b_2^{(0)}$ accounts for the Fermi statistics) and Δb_2 encapsulates the effects of the two-body interactions. The quantity $b_2^{(0)}$ can be obtained from the exact expression of the grand potential $\Omega^{(0)}$ of the non-interacting single-component Fermi gas [9],

$$\Omega^{(0)} = -V \frac{k_B T}{\lambda^3} \frac{2}{\sqrt{\pi}} \int_0^\infty x^{1/2} \ln(1 + ze^{-x}) dx. \quad (\text{S44})$$

Expanding the integrand and integrating term by term, $b_2^{(0)}$ can be shown to be equal to $\frac{-1}{2^{5/2}}$ [12]. The quantity Δb_2 can be written as $\Delta b_2 = (Q_2 - Q_2^{(0)})/Q_1$, where the superscript (0) refers, again, to the non-interacting system. The one-body partition function Q_1 , which appears in the denominator of the expression for Δb_2 , can be straightforwardly calculated:

$$Q_1 = \frac{1}{h^3} \int d^3 r d^3 p \exp\left(-\frac{p^2}{2mk_B T}\right) = \frac{V}{\lambda^3}, \quad (\text{S45})$$

where $\lambda = \sqrt{2\pi\hbar^2/(mk_B T)}$ is the thermal de Broglie wavelength. To evaluate $Q_2 - Q_2^{(0)}$, we use that the difference between the interacting and non-interacting partition functions is due to the relative motion (the center of mass motions are identical). This leads to the simplification

$$Q_2 - Q_2^{(0)} = \left[\frac{1}{h^3} \int d^3R d^3P \exp\left(-\frac{P^2}{4mk_B T}\right) \right] \left[\sum_{l,m} \int dk \left(g_{l,m}(k) - g_{l,m}^{(0)}(k) \right) \exp\left(\frac{\hbar^2 k^2}{mk_B T}\right) \right], \quad (\text{S46})$$

where \mathbf{R} and \mathbf{P} denote the two-body center of mass position and momentum vectors, respectively. The first square bracket on the right hand side of Eq. (S46) is the partition function of the center-of-mass motion. It evaluates to $2^{3/2}V/\lambda^3$. The second square bracket on the right hand side of Eq. (S46) is the partition function of the relative motion. The quantities $g_{l,m}(k)$ and $g_{l,m}^{(0)}(k)$ are the densities of states of the interacting and non-interacting two-body systems with relative orbital angular momentum l and associated projection quantum number m . For polarized fermions, l is restricted to odd values by the exchange symmetry. Since we focus on pure p -wave interactions in this work, $g_{l,m}(k) = g_{l,m}^{(0)}(k)$ for $l \geq 3$. Furthermore, the assumption of isotropic two-body interactions eliminates the dependence of $g_{l,m}(k)$ on m and the summation over m merely results in a multiplicative factor of 3 for $l = 1$.

To calculate $g_{l=1}(k) - g_{l=1}^{(0)}(k)$, a hard sphere potential of radius r_0 is assumed. Such a potential imposes a boundary condition on the relative two-body wave function [13]. Using that the asymptotic p -wave scattering wave function reads [14]

$$\psi_m(\mathbf{r}) \xrightarrow{r \rightarrow \infty} AY_{1,m}(\hat{\mathbf{r}}) \frac{\cos(kr + \delta_p(k))}{r}, \quad (\text{S47})$$

the energy spectrum $\epsilon_n(k) = \hbar^2 k_n^2/m$ of the hard sphere potential is determined by

$$k_n r_0 + \delta_p(k_n) = \frac{\pi}{2} + n\pi, \text{ where } n = 0, 1, 2, \dots \quad (\text{S48})$$

Therefore, when n increases by 1, k changes by $\Delta k = k_{n+1} - k_n$:

$$\Delta k r_0 + \delta_p(k_{n+1}) - \delta_p(k_n) = \pi. \quad (\text{S49})$$

Dividing both sides by Δk and taking the limit $r_0 \rightarrow 0$, one has $\Delta k \rightarrow dk$ and the above equation gives

$$r_0 + \frac{d\delta_p(k)}{dk} = \frac{\pi}{dk}. \quad (\text{S50})$$

Correspondingly, $\sum_n 1 \rightarrow \int dk g_{l=1}(k)$ and thus

$$g_{l=1}(k) - g_{l=1}^{(0)}(k) = \frac{1}{\Delta k} - \frac{1}{\Delta k^{(0)}} = \frac{1}{\pi} \frac{d\delta_p(k)}{dk}. \quad (\text{S51})$$

Combining Eqs. (S43), (S45), (S46) and Eq. (S51), we obtain the Beth-Uhlenbeck formula

$$b_2 = b_2^{(0)} + \frac{3 \times 2^{3/2}}{\pi} \int_0^\infty dk \exp\left(-\frac{\hbar^2 k^2}{mk_B T}\right) \frac{d\delta_p(k)}{dk}. \quad (\text{S52})$$

To perform the integral in Eq. (S52), we expand $d\delta_p(k)/dk$ with the help of Eq. (S1),

$$\frac{d\delta_p(k)}{dk} = \frac{d\delta_p(k)}{d \tan(\delta_p(k))} \frac{d \tan(\delta_p(k))}{dk} = -3v_p k^2 + \frac{5v_p^2}{R} k^4. \quad (\text{S53})$$

Substituting Eq. (S53) into Eq. (S52), b_2 is expressed in terms of v_p and R ,

$$b_2 = -\frac{1}{4\sqrt{2}} + \frac{18\pi v_p (5\pi v_p - \lambda^2 R)}{\lambda^5 R}. \quad (\text{S54})$$

Together with Eqs. (S42) and (S45), Eq. (S54) provides an explicit expression for the grand potential Ω . To determine explicit expressions for the p -wave contacts, the grand potential needs to be reexpressed in terms of the Helmholtz

free energy. To accomplish this task, the fugacity z needs to be expressed in terms of $n = N/V$. By the Gibbs-Duhem relation [15], one has

$$N = - \frac{z}{k_B T} \left. \frac{\partial \Omega}{\partial z} \right|_{V, T}. \quad (\text{S55})$$

Using $\Omega = -PV$, where P denotes the pressure, Eq. (S55) can be rewritten as

$$n = \frac{z}{k_B T} \left. \frac{\partial P}{\partial z} \right|_T. \quad (\text{S56})$$

Using the expansion of Ω in terms of z , we find the following expansions for the pressure and density:

$$P = \frac{k_B T z (1 + b_2 z)}{\lambda^3} \quad (\text{S57})$$

and

$$n = \frac{z}{\lambda^3} + 2b_2 \frac{z^2}{\lambda^3}. \quad (\text{S58})$$

At this order of the expansion, an explicit expression for z can be obtained from the algebraic equation Eq. (S58). The physical solution reads

$$z = \frac{-1}{4b_2} + \frac{\sqrt{1 + 8b_2 \lambda^3 n}}{4b_2}. \quad (\text{S59})$$

The free energy F can then be calculated using $F = \Omega + \mu N = \Omega + nV k_B T \ln(z)$ and using the right hand side of Eq. (S59) to eliminate z from $\Omega + nV k_B T \ln(z)$; as a result, F is expressed in terms of n . With this in hand, expressions for the two p -wave contacts can be derived in terms of n , $\Re(v_p)$, and $\Re(R)$. To this end, we Taylor-expand the contacts—consistent with the virial expansion framework applied to the thermodynamic quantities—up to second order in $n\lambda^3$:

$$C_v = \frac{2m(\Re(v_p))^2}{3\hbar^2} \left. \frac{\partial F}{\partial v_p} \right|_R = \frac{12\pi m k_B T n^2 V (\Re(v_p))^2}{\hbar^2} + \frac{60\pi m^2 k_B^2 T^2 n^2 V (\Re(v_p))^3}{\Re(R)\hbar^4}, \quad (\text{S60})$$

$$C_R = \frac{2m(\Re(R))^2}{3\hbar^2} \left. \frac{\partial F}{\partial R} \right|_{v_p} = \frac{30\pi m^2 k_B^2 T^2 n^2 V (\Re(v_p))^2}{\hbar^4}. \quad (\text{S61})$$

We emphasize that the expressions for the two p -wave contacts in Eqs. (S60) and (S61) apply to any p -wave system that is characterized by v_p and R , including systems with arbitrarily large $|\Re(v_p)|$ and $|\Re(R)|$. Specifically, the assumption of weak interactions has not yet entered into the derivation. Importantly, in addition to the expected linear dependence of C_v on T , C_v and C_R each contain a term that depends quadratically on T . Both these quadratic terms arise from the effective range: The T^2 term in C_v explicitly depends on the effective range while the T^2 in C_R arises from taking the derivative of F with respect to R . Equations (S60) and (S61) are identical to the expressions reported in Ref. [5]; in that work, it was suspected that the two T^2 terms may account for the suppression of β/T . In this context, though, it is important to keep in mind that the expressions for C_v and C_R , Eqs. (S60) and (S61), are derived within the high-temperature virial expansion approach while the suppression was experimentally observed in the low-temperature regime [16].

We now consider the weakly-interacting regime by taking the limit of Eq. (S2). Technically, we first express $\Re(R)$ to be $c_1(\Re(v_p))^{1/3}$, where c_1 is a constant of order one, and then evaluate Eqs. (S60) and (S61) in the regime where $n\Re(v_p) \ll 1$. The results are

$$C_v = \frac{12\pi m k_B T n^2 V (\Re(v_p))^2}{\hbar^2}, \quad (\text{S62})$$

$$C_R = \frac{30\pi m^2 k_B^2 T^2 n^2 V (\Re(v_p))^2}{\hbar^4}. \quad (\text{S63})$$

Substituting Eqs. (S62) and (S63) into Eq. (S41), dN/dt and subsequently β can be found. The result can be further simplified by repeating the progress above for the imaginary parts: expressing $\Im(R)$ as $c_2(\Im(v_p))^{1/3}$, where c_2 is a real number, and taking the limit $n\Im(v_p) \ll 1$, the term that includes R disappears.

Harmonically trapped system.— As demonstrated by Eq. (S43), b_2 is determined by Q_n with $n = 1$ and 2 . This implies that one can get b_2 by solving the one- and two-body problems. Busch *et al.* [17] reported the eigen states and eigen energies for two s -wave particles with zero-range pseudo-potential [18] confined in a harmonic trap. The approach was subsequently generalized to two spin-polarized fermions in a harmonic trap interacting through a p -wave pseudo-potential [19]. The energy spectrum for the relative motion was shown to be implicitly given by

$$\frac{v_p}{a_{\text{ho}}^3} \left(1 - \frac{2mv_p E_{\text{rel}}}{\hbar^2 R} \right) = - \frac{\Gamma \left(-\frac{E_{\text{rel}}}{2\hbar\omega} - \frac{1}{4} \right)}{2\sqrt{2}\Gamma \left(-\frac{E_{\text{rel}}}{2\hbar\omega} + \frac{5}{4} \right)}, \quad (\text{S64})$$

where ω and $a_{\text{ho}} = \sqrt{\hbar/(m\omega)}$ are the angular frequency of the harmonic trap and the length scale determined by it. In writing Eq. (S64), the phase shift is expressed using the effective range expansion. Similar to the discussion above, when $v_p \rightarrow 0$, the second term in round brackets on the left hand side of Eq. (S64) scales as $|v_p/R|a_{\text{ho}}^2 \propto |v_p^{2/3}|/a_{\text{ho}}^2 < |v_p^{2/3}|n^{2/3} \ll 1$. Dropping the energy-dependent term on the left hand side, Eq. (S64) reduces to

$$\frac{v_p}{a_{\text{ho}}^3} = - \frac{\Gamma \left(-\frac{E_{\text{rel}}}{2\hbar\omega} - \frac{1}{4} \right)}{2\sqrt{2}\Gamma \left(-\frac{E_{\text{rel}}}{2\hbar\omega} + \frac{5}{4} \right)}. \quad (\text{S65})$$

When v_p vanishes identically, Eq. (S65) recovers the p -wave energy spectrum of the non-interacting system [19]:

$$E_{\text{rel}}^{(0)}(n, l = 1) = \left(2n + \frac{5}{2} \right) \hbar\omega \quad n = 0, 1, 2, \dots \quad (\text{S66})$$

The energy spectrum of the weakly-interacting system can thus be obtained by Taylor-expanding Eq. (S65) in v_p/a_{ho}^3 around the non-interacting relative energies,

$$E_{\text{rel}}(n, 1) = E_{\text{rel}}^{(0)}(n, 1) + f(n) \frac{v_p}{a_{\text{ho}}^3}, \quad (\text{S67})$$

where $f(n)$ is given by $a_{\text{ho}}^3 dE_{\text{rel}}/dv_p$, evaluated at $E_{\text{rel}}^{(0)}(n, 1)$ with $n = 0, 1, 2, \dots$,

$$f(n) = \lim_{E_{\text{rel}} \rightarrow (5/2+2n)\hbar\omega} a_{\text{ho}}^3 \left(\frac{dv_p}{dE_{\text{rel}}} \right)^{-1} = \frac{4\sqrt{2}}{(-1)^n n! \Gamma \left(-\frac{3}{2} - n \right)} \hbar\omega, \quad \text{where } n = 0, 1, 2, \dots \quad (\text{S68})$$

The two-body partition function Q_2 is then

$$Q_2 = Q_1 \left\{ \sum_{n=0}^{\infty} \left[3 \exp \left(-\frac{E_{\text{rel}}(n, 1)}{k_B T} \right) + \sum_{l=3,5,7,\dots} (2l+1) \exp \left(-\frac{E_{\text{rel}}^{(0)}(n, l)}{k_B T} \right) \right] \right\}, \quad (\text{S69})$$

where $E_{\text{rel}}^{(0)}(n, l) = (2n + l + 3/2)\hbar\omega$ is the non-interacting energy spectrum for the l th partial wave channel. The Q_1 in Eq. (S69) is the partition function of the center-of-mass motion. Since the center-of-mass motion is identical to that of a non-interacting particle, its partition function is equal to the one-body partition function:

$$Q_1 = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} (2l+1) \exp \left(-\frac{E_{\text{rel}}^{(0)}(n, l)}{k_B T} \right) = \frac{\exp \left(\frac{3\hbar\omega}{2k_B T} \right)}{\left(\exp \left(\frac{\hbar\omega}{k_B T} \right) - 1 \right)^3}. \quad (\text{S70})$$

The expression inside the curly bracket on the right hand side of Eq. (S69) represents the partition function of the relative motion. The summation only goes over odd relative orbital angular momentum quantum numbers l , since the spatial wave function of two polarized fermions (two fermions in the same spin state) must be anti-symmetric under

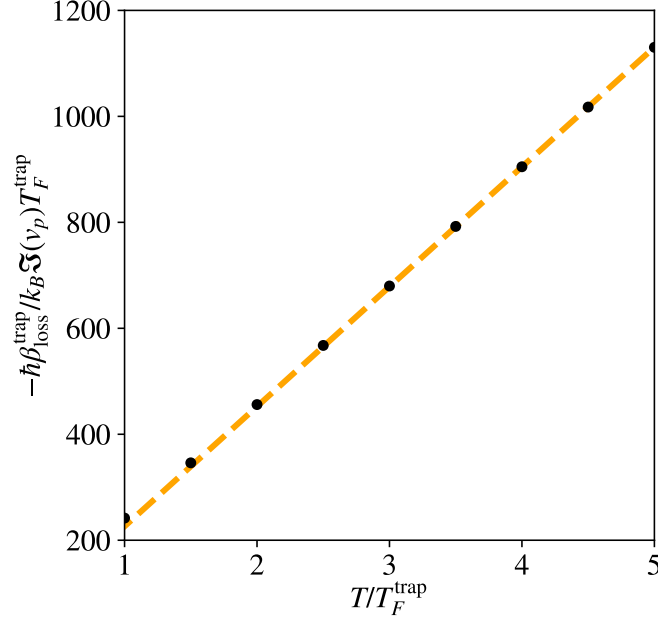


FIG. S1. $\beta_{\text{loss}}^{\text{trap}}$ against T in the high- T regime. The dashed line and the black circles show Eq. (7) from the main text and the prediction from the second-order virial expansion based on the exact two-body energy spectrum [19].

the exchange of the two particles. The sum $\sum_{l=3,5,\dots}$ comes from the higher angular momentum states, which are by assumption not impacted by the interactions. The sum has a compact analytical expression:

$$\sum_{l=3,5,7,\dots} (2l+1) \exp\left(-\frac{E_{\text{rel}}^{(0)}(n,l)}{k_B T}\right) = \exp\left(\frac{-\hbar\omega}{2k_B T}\right) \frac{\left[-3 + 7 \exp\left(\frac{2\hbar\omega}{k_B T}\right)\right]}{\left[\exp\left(\frac{2\hbar\omega}{k_B T}\right) - 1\right]^3}. \quad (\text{S71})$$

The first term in the curly brackets on the right hand side of Eq. (S69) corresponds to p -wave states (the factor of 3 is due due to the three projection quantum numbers $m = -1, 0, 1$). The sum is evaluated numerically by choosing a sufficiently large energy cutoff (i.e., maximal n value).

With b_2 evaluated, the subsequent steps proceed analogously to those for the homogeneous system. Specifically, z needs to be converted to N by solving Eq. (S55). The physical solution is

$$z = \frac{Q_1 - \sqrt{Q_1^2 - 4NQ_1^2 + 8NQ_2}}{2(Q_1^2 - 2Q_2)}. \quad (\text{S72})$$

Substituting Eq. (S72) into Eq. (S42), we get

$$\Omega = k_B T \frac{4NQ_2 + Q_1 \left[-(2N+1)Q_1 + \sqrt{Q_1^2 - 4NQ_1^2 + 8NQ_2} \right]}{4(Q_1^2 - 2Q_2)}. \quad (\text{S73})$$

Using $F = \Omega + \mu N$, C_v is given by

$$C_v = \frac{2m}{3\hbar^2} \frac{\partial F(\Re(v_p))}{\partial v_p} (\Re(v_p))^2 = -\frac{mk_B T (\Re(v_p))^2}{3\hbar^2} \frac{4NQ_2 - Q_1 \left((2N-1)Q_1 + \sqrt{Q_1^2 - 4NQ_1^2 + 8NQ_2} \right)}{(Q_1^2 - 2Q_2)^2} \frac{\partial Q_2(\Re(v_p))}{\partial v_p}. \quad (\text{S74})$$

It follows that dN/dt for the harmonically trapped system can be evaluated using Eq. (2). This allows one to calculate $\beta_{\text{loss}}^{\text{trap}}$ [Eq. (7) in the main text] directly without resorting to the local density approximation(LDA). Figure S1 shows excellent agreement between the black dots (virial expansion for the trapped system) and the dashed line [Eq. (7) in the main text]. The result shown in Fig. S1 uses an energy cutoff of $4,000\hbar\omega$. The agreement provides an important validation of LDA framework applied in the main text.

III. LANDAU FERMI LIQUID THEORY

Our derivation builds on a recent work, which developed a description of the weakly-interacting homogeneous polarized Fermi gas using Fermi liquid theory [20]. Here, their formalism is directly extended to the complex domain because, under the condition Eq. (S2), the second-order expansion of all thermodynamic quantities and Fermi liquid parameters are automatically analytic. The ground state energy E_0 of the homogeneous p -wave gas is given by

$$E_0 = NE_F \left(\frac{3}{5} + \frac{6k_F^3 v_p}{5\pi} - \left(\frac{18}{35\pi k_F R} - \frac{2066 - 312 \ln(2)}{1155\pi^2} \right) k_F^6 v_p^2 \right), \quad (\text{S75})$$

where $E_F = \hbar^2 k_F^2 / 2m$ and k_F denote the Fermi energy and Fermi momentum, respectively, of the non-interacting system. To obtain the temperature dependence of the internal energy, one can integrate the heat capacity $c_V(T)$,

$$U_{\text{int}} = E_0 + V \int_0^T c_V(T') dT' = E_0 + \frac{\pi^2}{12} \nu(0) V k_B^2 T^2, \quad (\text{S76})$$

where [21]

$$c_V(T) = \frac{\pi^2}{6} \nu(0) k_B^2 T. \quad (\text{S77})$$

Here, $\nu(0)$ is the density of states at the Fermi surface, which is determined by the effective mass m^* and the Fermi wave vector k_F ,

$$\nu(0) = \frac{m^* k_F}{2\pi^2 \hbar^2}. \quad (\text{S78})$$

The effective mass reads [20]

$$\frac{m}{m^*} = 1 + \frac{2k_F^3}{\pi} v_p + \left(\frac{2}{\pi k_F R} - \frac{8(313 - 426 \ln(2))}{315\pi^2} \right) k_F^6 v_p^2. \quad (\text{S79})$$

Since our aim is to derive expressions for the p -wave contacts by taking partial derivatives of the Helmholtz free energy, we need to “switch” from “energy” to “Helmholtz free energy.” This can be accomplished via the relation $F = U_{\text{int}} - TS$, where S denotes the entropy. We start with the definition of the entropy

$$\begin{aligned} S &= -k_B \sum_{\mathbf{p}} [n_{\mathbf{p}} \ln n_{\mathbf{p}} + (1 - n_{\mathbf{p}}) \ln (1 - n_{\mathbf{p}})] \\ &= \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{+\infty} dp p^2 n_p \ln \left(\frac{1 - n_p}{n_p} \right) - \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{+\infty} dp p^2 \ln (1 - n_p), \end{aligned} \quad (\text{S80})$$

where $n_{\mathbf{p}}$ is the quasi-particle momentum distribution function. In the last equal sign of the last equation, we assumed a spherically symmetric quasi-particle momentum distribution function and converted the sum over \mathbf{p} to an integral. Changing the variable p to the energy ϵ_p using the relation $d\epsilon_p = (p/m^*) dp$, we find

$$S = \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{+\infty} d\epsilon_p m^* p n_p \ln \left(\frac{1 - n_p}{n_p} \right) - \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{+\infty} d\epsilon_p m^* p \ln (1 - n_p). \quad (\text{S81})$$

Applying the Sommerfeld expansion to the first term of the above expression, we find

$$S = \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{\mu} d\epsilon_p m^* p \ln \left(\frac{1 - n_p}{n_p} \right) + \frac{V m^* p_F}{12 \hbar^3} k_B^2 T - \frac{k_B V}{2\pi^2 \hbar^3} \int_0^{+\infty} d\epsilon_p m^* p \ln (1 - n_p). \quad (\text{S82})$$

In the weakly-interacting limit ($n|v_p| \ll 1$), the quasi-particle residue would approach zero. Thus, as $T \ll T_F$, we have $n_p \simeq 1$ for $p < p_F$ and $n_p \simeq 0$ for $p > p_F$. Hence, the first term and the third term above can be canceled to get

$$\begin{aligned} S &= \frac{V m^* p_F}{12 \hbar^3} k_B^2 T \\ &= \frac{\pi^2}{6} \nu(0) V k_B^2 T. \end{aligned} \quad (\text{S83})$$

Inserting the above result for S , the Helmholtz free energy of the system becomes

$$F = U_{\text{int}} - TS = E_0 - \frac{\pi^2}{12} \nu(0) V k_B^2 T^2. \quad (\text{S84})$$

The two p -wave contacts can now be obtained by taking partial derivatives of the free energy. We find

$$C_v = \frac{12 \times 6^{2/3} \pi^{7/3} n^{8/3} V (\Re(v_p))^2}{5} + \frac{2^{1/3} \pi^{5/3} m^2 k_B^2 T^2 n^{4/3} V (\Re(v_p))^2}{3^{2/3} \hbar^4}, \quad (\text{S85})$$

$$C_R = \frac{216 \times 6^{1/3} \pi^{11/3} n^{10/3} V (\Re(v_p))^2}{35} + \frac{2\pi^3 m^2 k_B^2 T^2 n^2 V (\Re(v_p))^2}{\hbar^4}. \quad (\text{S86})$$

Note that both Eqs. (S85) and (S86) are calculated up to the second order in $n\Re(v_p)$, after restricting $\Re(R) \propto (\Re(v_p))^{1/3}$. Similar to the case at high temperatures, the contribution to dN/dt and β from C_R can be discarded by taking $n\Im(v_p) \ll 1$ after substituting Eqs. (S85) and (S86) into Eq. (S41).

It is worth noting that the T^2 terms from Eqs. (S85) propagate into the expression for β ; they do not—as in Eq. (S62), which is applicable to the weakly interacting regime,—become negligible. In this sense, Ref. [5] indeed gives the correct insight that a T^2 term might play a role in C_v . We emphasize, however, that the mechanisms behind the T^2 terms in Eqs. (S85) and (S62) are completely different than those considered in Ref.[5].

IV. LOCAL-DENSITY APPROXIMATION

The phase space density of the non-interacting homogeneous single-component Fermi gas follows the Fermi-Dirac distribution

$$w(\mathbf{r}, \mathbf{p}) = \frac{1}{(2\pi)^3} \frac{1}{\exp[(p^2/2m - \mu)/k_B T] + 1}. \quad (\text{S87})$$

Using this, the phase space density of the inhomogeneous system (isotropic harmonic trap with angular trap frequency ω) can be obtained within the Thomas-Fermi approximation or LDA [22]:

$$\begin{aligned} w(\mathbf{r}, \mathbf{p}) &= \frac{1}{(2\pi)^3} \frac{1}{\exp[(p^2/2m - \mu(\mathbf{r}))/k_B T] + 1} \\ &= \frac{1}{(2\pi)^3} \frac{1}{\exp[(p^2/2m + m\omega^2 r^2/2 - \mu)/k_B T] + 1}. \end{aligned} \quad (\text{S88})$$

The real space density is obtained by integrating out the momentum dependence:

$$\begin{aligned} n(\mathbf{r}) &= \frac{1}{(2\pi\hbar)^3} \int d^3p \frac{1}{\exp[(p^2/2m + m\omega^2 r^2/2 - \mu)/k_B T] + 1} \\ &= - \frac{(mk_B T)^{3/2} \text{Li}_{3/2} \left[-\exp\left(\frac{2\mu - m\omega^2 r^2}{2k_B T}\right) \right]}{(2\pi)^{3/2} \hbar^3}, \end{aligned} \quad (\text{S89})$$

where Li denotes the polylog function. The chemical potential μ is determined by the restriction on the total number of particles N ,

$$N = \int d\epsilon \frac{g(\epsilon)}{\exp[(\epsilon - \mu)/k_B T] + 1}, \quad (\text{S90})$$

where $g(\epsilon) = \epsilon^2/[2(\hbar\omega)^3]$ is the density of states [22]. The explicit form of μ is

$$\mu = k_B T \ln \left[-\text{Li}_3^{-1} \left(-\frac{(\hbar\omega)^3 N}{(k_B T)^3} \right) \right]. \quad (\text{S91})$$

Li_3^{-1} represents the inverse function of Li_3 , i.e. $y = f^{-1}(x)$ indicates $x = f(y)$.

In practice, the $T \simeq 0$ case needs special attention due to the difficulty of evaluating Eq. (S89) numerically. In this case, one can start from the chemical potential, which is—at $T = 0$ —equal to the Fermi energy:

$$\mu = E_F = (6N)^{1/3} \hbar\omega. \quad (\text{S92})$$

In the homogeneous system, the relationship between the chemical potential of the ground state and the density is

$$n = \frac{1}{6\pi^2} \left(\frac{2m\mu}{\hbar^2} \right)^{3/2}. \quad (\text{S93})$$

Using the LDA, the density profile of the trapped system at $T = 0$ is

$$n(\mathbf{r}) = \frac{1}{6\pi^2} \left(\frac{48^{1/3} N^{1/3} a_{\text{ho}}^2 - r^2}{a_{\text{ho}}^4} \right)^{3/2} \quad \text{for } r < R_F = (48N)^{1/6} a_{\text{ho}}. \quad (\text{S94})$$

The cutoff or Thomas-Fermi radius R_F is determined by the condition $n(\mathbf{r}) \geq 0$.

With the expressions for $n(\mathbf{r})$ given above (either the $T > 0$ or the $T = 0$ expression), $\beta_{\text{loss}}^{\text{trap}}$ can be calculated using Eq. (7) from the main text.

Interpreting n^{trap} to be a functional of $n(\mathbf{r})$, β^{trap} can be calculated directly within the LDA. We start with the definition of n^{trap} [see Table (S1)]:

$$n^{\text{trap}}[n(\mathbf{r})] = \frac{\int d^3r [n(\mathbf{r})]^2}{\int d^3r n(\mathbf{r})}. \quad (\text{S95})$$

The functional derivative and functional differential of n^{trap} are

$$\frac{\delta n^{\text{trap}}}{\delta n(\mathbf{r})} = \frac{2n(\mathbf{r})}{\int d^3r n(\mathbf{r})} - \frac{\int d^3r [n(\mathbf{r})]^2}{(\int d^3r n(\mathbf{r}))^2}, \quad (\text{S96})$$

$$dn^{\text{trap}} = \int d^3r \frac{\delta n^{\text{trap}}}{\delta n(\mathbf{r})} \delta n(\mathbf{r}). \quad (\text{S97})$$

The variation of $n(\mathbf{r})$ is then governed by the homogeneous two-body loss coefficient $\beta(\mathbf{r})$, which is defined by

$$\delta n(\mathbf{r}) = -\beta(\mathbf{r}) [n(\mathbf{r})]^2 dt, \quad (\text{S98})$$

where $\beta(\mathbf{r})$ is interpreted to be the *homogeneous* β evaluated with a local Fermi temperature

$$T_F(\mathbf{r}) = \frac{\hbar^2}{2mk_B} [6\pi^2 n(\mathbf{r})]^{2/3}. \quad (\text{S99})$$

For any \mathbf{r} , with Eqs. (S89), (S99) and the result reported in Fig. 1 of the main text, $\beta(\mathbf{r})$ can be evaluated. Substituting Eq. (S97) into the definition of β^{trap} [see Table (S1)], we obtain

$$\beta^{\text{trap}} = \frac{2 (\int d^3r n(\mathbf{r})) (\int d^3r \beta(\mathbf{r}) [n(\mathbf{r})]^3)}{(\int d^3r [n(\mathbf{r})]^2)^2} - \frac{\int d^3r \beta(\mathbf{r}) [n(\mathbf{r})]^2}{\int d^3r [n(\mathbf{r})]^2}. \quad (\text{S100})$$

Comparing with the definitions of β^{trap} and $\beta_{\text{loss}}^{\text{trap}}$, the excessive part that is due to the change of the volume is identified as

$$\beta_{\text{deform}}^{\text{trap}} = \frac{2 (\int d^3r n(\mathbf{r})) (\int d^3r \beta(\mathbf{r}) [n(\mathbf{r})]^3)}{(\int d^3r [n(\mathbf{r})]^2)^2} - \frac{2 \int d^3r \beta(\mathbf{r}) [n(\mathbf{r})]^2}{\int d^3r [n(\mathbf{r})]^2}. \quad (\text{S101})$$

When $\beta(\mathbf{r})$ becomes independent of \mathbf{r} , which is the case in the high-temperature regime, β^{trap} and $\beta_{\text{deform}}^{\text{trap}}$ are proportional to the loss coefficient β of the homogeneous system. At high temperature, Eq. (S89) reduces to

$$n(\mathbf{r}) = \frac{m^{3/2} \omega^3 N \exp\left(\frac{-m\omega^2 r^2}{2k_B T}\right)}{(2\pi k_B T)^{3/2}}. \quad (\text{S102})$$

In this case, the integrals in Eq. (S100) and Eq. (S101) can be explicitly performed. The results are

$$\beta^{\text{trap}} = \left(\frac{16}{3\sqrt{3}} - 1 \right) \beta, \quad (\text{S103})$$

$$\beta_{\text{deform}}^{\text{trap}} = \left(\frac{16}{3\sqrt{3}} - 2 \right) \beta. \quad (\text{S104})$$

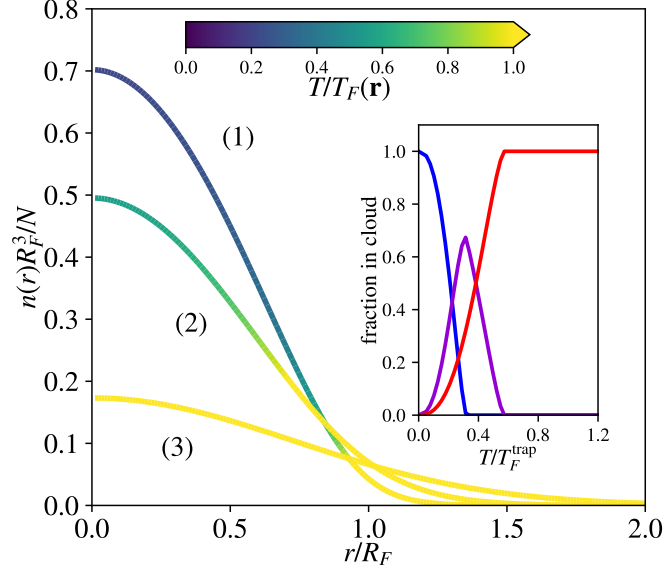


FIG. S2. Density profiles for $T/T_F^{\text{trap}} = 0.2, 0.4,$ and 1 [curves labeled by (1), (2), and (3), respectively; $R_F = (48N)^{1/6} \sqrt{\hbar/m\bar{\omega}}$ is the radius of the $T = 0$ cloud]. Inset: The blue, purple, and red lines show the fraction of the inhomogeneous loss-rate coefficient due to the high-, medium-, and low-temperature theory of the homogeneous system as a function of the local temperature.

There is an illuminating way of writing Eqs. (S100) and (S101) at high temperatures. Using $\langle \dots \rangle$ to denote an average over the density profile [$\langle \dots \rangle = N^{-1} \int d^3r \dots n(\mathbf{r})$], the equations can be rearranged into

$$\beta^{\text{trap}} \xrightarrow{T/T_F \gg 1} \beta \left(\frac{2\langle n^2(\mathbf{r}) \rangle - \langle n(\mathbf{r}) \rangle^2}{\langle n(\mathbf{r}) \rangle^2} \right), \quad (\text{S105})$$

$$\beta_{\text{deform}}^{\text{trap}} \xrightarrow{T/T_F \gg 1} 2\beta \left(\frac{\langle n^2(\mathbf{r}) \rangle - \langle n(\mathbf{r}) \rangle^2}{\langle n(\mathbf{r}) \rangle^2} \right). \quad (\text{S106})$$

Equation (S106) indicates that $\beta_{\text{deform}}^{\text{trap}}$ is proportional to the variance of the local density of the system at high temperatures. It should be emphasized that the variance of the local density is not equivalent to the “density fluctuations” discussed in Ref. [16]; the latter are particle fluctuations *at fixed chemical potential* of the homogeneous system. In this work, as we used the Helmholtz free energy to define the contact within the canonical ensemble, the number of particles is fixed. Therefore, our picture is different from the conjecture that density fluctuations suppress the loss proposed in Ref. [16].

The lines labeled (1), (2), and (3) in Fig. S2 show the density profile for a spatially symmetry trap with angular frequency $\bar{\omega}$ for $T/T_F^{\text{trap}} = 0.2, 0.4,$ and 1 , respectively. The colorcoding of the curves indicates the local dimensionless temperature $T/T_F(\mathbf{r})$. It can be seen that within the frame of LDA, for the case of quantum degeneracy in the trap where $T/T_F^{\text{trap}} \lesssim 1$, $T/T_F(\mathbf{r})$ is still larger than 1 for the majority of the density profile. As demonstrated by the inset of Fig. S2 and curve (2) of Fig. S2, even for $T/T_F^{\text{trap}} = 0.4$, the “hot part” of the cloud extends to r/R_F as small as ~ 0.6 . The inset shows that about half of the cloud is captured by the high-temperature equation of state and the other half by the intermediate-temperature equation of state (purple line). For $T/T_F(\mathbf{r}) = 0.2$, in contrast, the low-temperature equation of state (blue line) contributes about 30%.

V. REPRODUCTION OF THE EXPERIMENTAL RESULTS

This section establishes an explicit connection between β^{trap} and β_{exp} . The main text already introduced that the analysis of the experimental data approximates the in-situ average density by [16]:

$$n_{\text{exp}}^{\text{trap}}(T) = \frac{N}{V_{\text{exp}}^{\text{trap}}} = \frac{N}{8\pi^{3/2}} \bar{\omega}^3 \left(\frac{k_B T}{m} \right)^{-3/2}, \quad (\text{S107})$$

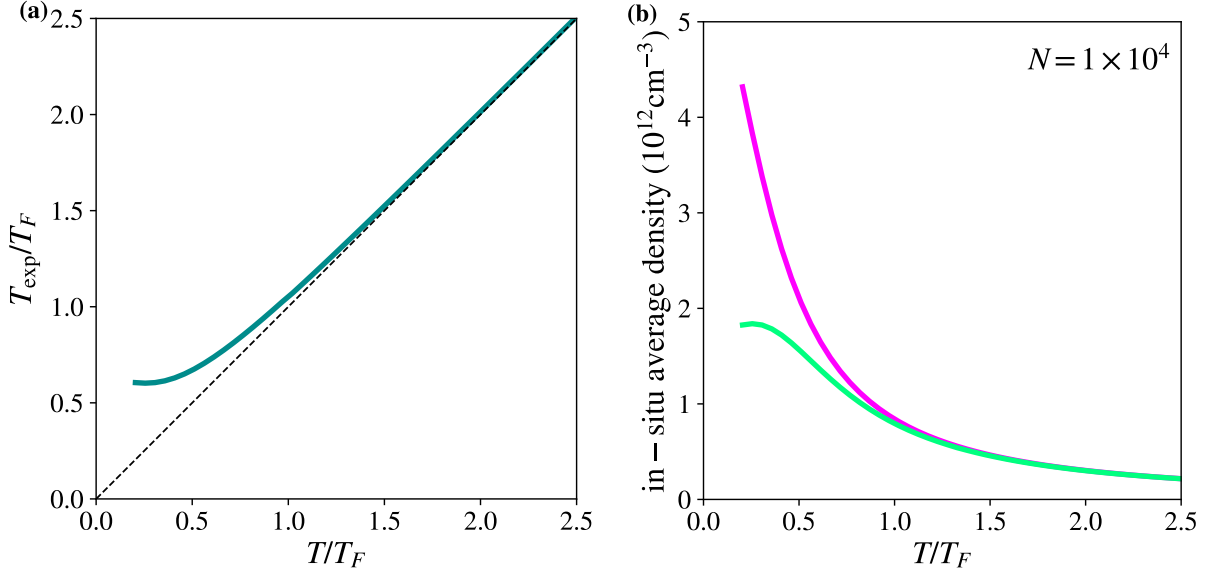


FIG. S3. (a) Comparison between "expansion temperature" T_{exp} and T scaled by T_F . The relation does not depend on the total number of particles N in the system. The dashed black line shows $T_{\text{exp}} = T$ to guide the eye. (b) In-situ average densities against T/T_F . Green line represents $n_{\text{exp}}^{\text{trap}}$ and magenta line is n^{trap} . N is set to be 1×10^4 for this plot.

where $\bar{\omega}$ is the geometric mean of the angular frequencies in the x -, y -, and z -directions. In the following discussion, we replace the experimental trap by an isotropic trap with $\bar{\omega} = 479$ Hz. As can be seen by inserting Eq. (S102) into n^{trap} , Eq. (S107) is merely the high-temperature limit of it:

$$n^{\text{trap}} = \frac{1}{N} \int d^3r \left(\frac{m^{3/2} \omega^3 N \exp\left(\frac{-m\omega^2 r^2}{2k_B T}\right)}{(2\pi k_B T)^{3/2}} \right)^2 = \frac{N}{8\pi^{3/2}} \bar{\omega}^3 \left(\frac{k_B T}{m}\right)^{-3/2}. \quad (\text{S108})$$

Motivated by this, one might speculate that $n_{\text{exp}}^{\text{trap}}(T)$ does not provide a faithful description at lower temperatures and in the degenerate regime. However, in the experimental analysis, $n_{\text{exp}}^{\text{trap}}(T)$ is evaluated at the so-called "expansion temperature" T_{exp} , which was measured by fitting the density profile of the cloud after a long time of flight τ to the fitting function

$$n_{\text{fit}} = n_0 \exp\left(\frac{m\bar{\omega}^2 r^2}{2k_B T_{\text{exp}}(1 + (\bar{\omega}\tau)^2)}\right), \quad (\text{S109})$$

where n_0 and T_{exp} were treated as fitting parameters. To understand the behavior of T_{exp} , Fig. S3(a) shows T_{exp}/T_F as a function of T/T_F . To obtain T_{exp} , the experimental sequence is emulated. Specifically, the ballistic expansion is simulated numerically, starting with the confined equilibrated cloud with temperature T . The expanded cloud is then fit to Eq. (S109). For the calculations shown in Fig. S3, $\bar{\omega}\tau = 2\pi$ is used. We tested that this value is sufficiently large to obtain converged results. For $T \gg T_F$, T_{exp} obtained from the above approach should be the same as the physical temperature T [23]. Figure S3 shows that this is indeed the case for $T/T_F \gtrsim 1.2$. In the quantum degenerate regime, in contrast, T_{exp} is higher than the physical temperature T and approaches a constant as T approaches zero. Correspondingly, there exists a large deviation between $n_{\text{exp}}^{\text{trap}}$ and n^{trap} at low temperatures. This is demonstrated in Fig. S3(b).

The main text shows that the global loss coefficient β^{trap} , which characterizes the trapped system, contains two parts, namely $\beta_{\text{loss}}^{\text{trap}}$ and $\beta_{\text{deform}}^{\text{trap}}$. Correspondingly, the use of $n_{\text{trap}}^{\text{trap}}(T_{\text{exp}})$ also leads to two terms. Taking the derivative of both sides of Eq. (S107) with respect to t , we find

$$\frac{dn_{\text{exp}}^{\text{trap}}}{dt} = -\beta_{\text{exp}}(n_{\text{exp}}^{\text{trap}})^2 - \frac{3}{2} \frac{n_{\text{exp}}^{\text{trap}}}{T_{\text{exp}}} \frac{dT_{\text{exp}}}{dt}, \quad (\text{S110})$$

where we used

$$\beta_{\text{exp}} = -\frac{V_{\text{exp}}^{\text{trap}}}{N^2} \frac{dN}{dt}. \quad (\text{S111})$$

The second term on the right hand side of Eq. (S110) arises from the time dependence of $V_{\text{exp}}^{\text{trap}}$; using $n_{\text{exp}}^{\text{trap}}$, the time dependence is "converted" to a time dependence of T_{exp} . As the volume of the system expands, T_{exp} rises gradually. This was referred to as anti-evaporation in the experimental KRb papers [16, 24]. While it might be tempting—motivated by the form of the first term on the right hand side of Eq. (S110)—to identify β_{exp} with $\beta_{\text{loss}}^{\text{trap}}$, the β_{exp} extracted from experiment differs from $\beta_{\text{loss}}^{\text{trap}}$ since the analysis of the experimental data used approximations for the global density and temperature. Therefore, in order to use our predictions (namely, β^{trap}) to compare with the experiment, we need to find an expression for β_{exp} in terms of β^{trap} . This is achieved by solving Eq. (S110) for β_{exp} ,

$$\beta_{\text{exp}} = -\frac{1}{(n_{\text{exp}}^{\text{trap}})^2} \frac{dn_{\text{exp}}^{\text{trap}}}{dt} - \frac{3}{2} \frac{1}{n_{\text{exp}}^{\text{trap}} T_{\text{exp}}} \frac{dT_{\text{exp}}}{dt}, \quad (\text{S112})$$

and then rewriting terms to bring in β^{trap} ,

$$\beta_{\text{exp}} = -\left[\frac{(n_{\text{exp}}^{\text{trap}})^2}{(n_{\text{exp}}^{\text{trap}})^2} \frac{dn_{\text{exp}}^{\text{trap}}}{dn_{\text{trap}}}\right] \left[\frac{1}{(n_{\text{exp}}^{\text{trap}})^2} \frac{dn_{\text{exp}}^{\text{trap}}}{dt}\right] - \frac{3}{2} \left(\frac{T}{T_{\text{exp}}} \frac{dT_{\text{exp}}}{dT}\right) \left(\frac{1}{n_{\text{exp}}^{\text{trap}} T} \frac{dT}{dt}\right) \quad (\text{S113})$$

$$= \left[\frac{(n_{\text{exp}}^{\text{trap}})^2}{(n_{\text{exp}}^{\text{trap}})^2} \frac{dn_{\text{exp}}^{\text{trap}}}{dn_{\text{trap}}}\right] \beta^{\text{trap}} - \frac{3}{2} \left(\frac{1}{T_{\text{exp}}} \frac{dT_{\text{exp}}}{dT}\right) \left(\frac{1}{n_{\text{exp}}^{\text{trap}}} \frac{dT}{dt}\right). \quad (\text{S114})$$

To reproduce the β_{exp}/T as a function of the *initial* T/T_F from Ref. [16], all T , T_{exp} , and their differentiations are evaluated at $t = 0$, i.e., from the equilibrated system before any loss processes have occurred. In fact, this is the only regime in which Eq. (S114) firmly works. For larger t , the density profile may deviate from its shape in thermal equilibrium. Since cloud deformation effects may be smeared out during the ballistic expansion, in practice one can very likely still obtain T_{exp} (and possibly even " T/T_F ") by fitting the density profile to a polylog function. The extracted T , however, might not represent the true temperature due to the fact that temperature is not a well-defined quantity in a non-equilibrated system. Similarly, we want to emphasize that the quantity dT/dt , which enters the calculation, may not correspond to a true physical "heating rate" of the cloud. If the time scale for the system to thermalize is much longer than the period of observation, the quantity dT/dt loses its physical meaning after a certain time. Hence, it is suggested to interpret it as representing the volume change. In other words, when treating non-homogeneous systems, where the volume can change, our method serves as a time-dependent perturbation approach.

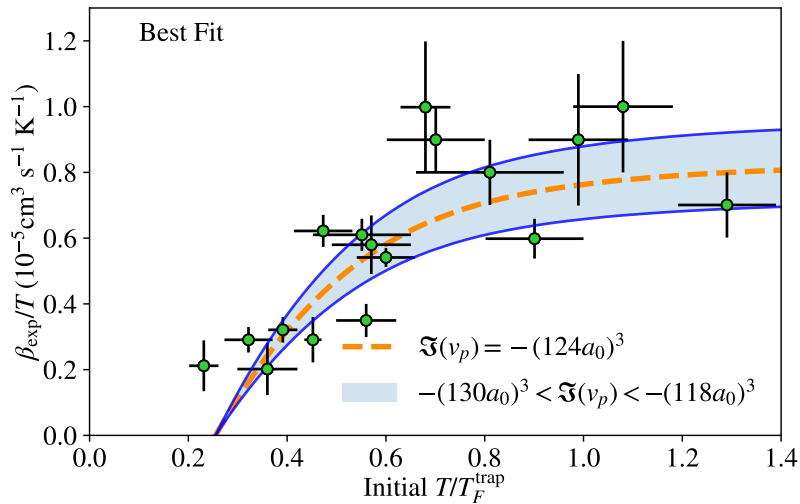


FIG. S4. Comparison between the experimental data from Ref. [16] and the theory predictions that use the imaginary part of the scattering volume obtained by fitting the experimental data of Ref. [16].

For our aim, at $t = 0$, dT/dt can be evaluated using Eq. (S101):

$$\frac{dT}{dt} = \frac{1}{N} \frac{dV^{\text{trap}}}{dt} \left(\frac{dn^{\text{trap}}}{dT} \right)^{-1} (n^{\text{trap}})^2 = \beta_{\text{deform}}^{\text{trap}} \left(\frac{dn^{\text{trap}}}{dT} \right)^{-1} (n^{\text{trap}})^2. \quad (\text{S115})$$

According to Fig. S3, T is very close to T_{exp} for $T/T_F \gtrsim 1.2$; this tells us that the experimentally measured β_{exp} coincides with $\beta_{\text{loss}}^{\text{trap}}$. Moreover, because $\beta = \beta_{\text{loss}}^{\text{trap}}$ for $T \gg T_F$ (see the main text), the high- T experiments explicitly yield the homogeneous loss coefficient β . Equation (4) of the main text expresses β in terms of the parameter $\Im(v_p)$. An earlier experiment [25] that operated in the high-temperature regime reported β/T to be $1.1(\pm 3) \times 10^{-5} \text{ cm}^3 \text{ s}^{-1} \text{ K}^{-1}$. Using the value of β/T , we extract $\Im(v_p) \simeq -(136_{-14}^{+11} a_0)^3$.

Alternatively and to further verify our results, we perform a non-linear fit to the experimental result of Ref. [16] using Eqs. (S114) and (S115). Weighting each experimental data point with $1/[(3\sigma_x)^2 + (3\sigma_y)^2]$, we obtain $\Im(v_p) = -(124_{-6}^{+6} a_0)^3$. This best fit value is consistent with both the predictions from the MQDT calculations and the value extracted from the high-temperature experiment. Figure (S4) shows the result.

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